Surface Tensions between Active Fluids and Solid Interfaces: bare vs dressed

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Abstract

We analyze the surface tension exerted at the interface between an active fluid and a solid boundary in terms of tangential forces. Focusing on active systems known to possess an equation of state for the pressure, we show that interfacial forces are of a more complex nature. Using a number of macroscopic setups, we show that the surface tension is a combination of an equation-of-state abiding part and of setup-dependent contributions. The latter arise from generic setup-dependent steady currents which "dress" the measurement of the "bare" surface tension. The former shares interesting properties with its equilibrium counterpart, and can be used to generalize the Young-Laplace law to active systems. We finally show how a suitably designed probe can directly access this bare surface tensions, which can also be computed using a generalized Virial formula.

Surface tension

Mechanics: Tangential force exerted on the particles at an interface.

Thermodynamics: Free energy required to create unit area/length of the interface.

Active Brownian particles (ABPs)

Self-propelled: Particles have "engines" to propel their motion.

Rotational diffusion: Particle orientation does Brownian motion.

Equation of motion (η Gaussian white noise with unit variance, $\mathbf{u}(\theta) = (\cos\theta, \sin\theta)$).

$$\dot{\mathbf{r}} = v_0 \mathbf{u}(\theta) - \nabla V_{\text{ext}}(\boldsymbol{r}) \qquad \dot{\theta} = \sqrt{2D_r}\eta$$

Passive vs active

Passive particles have equations of state (EOS). The pressure and surface tension are well-defined concept which can be measured in different ways and always lead to the same results, independently of the measurement setup.

Active particles in general do not have equations of state and do not minimize a free energy. The pressure and surface tension will depend on the detailed property of the container.

Spherical non-interacting ABPs have equation of state of their pressure. $P_b = v_0^2\rho_0/d\mu D_r$.

Question: Is there an analogous expression for the solid-gas surface tension?

Langmuir setup

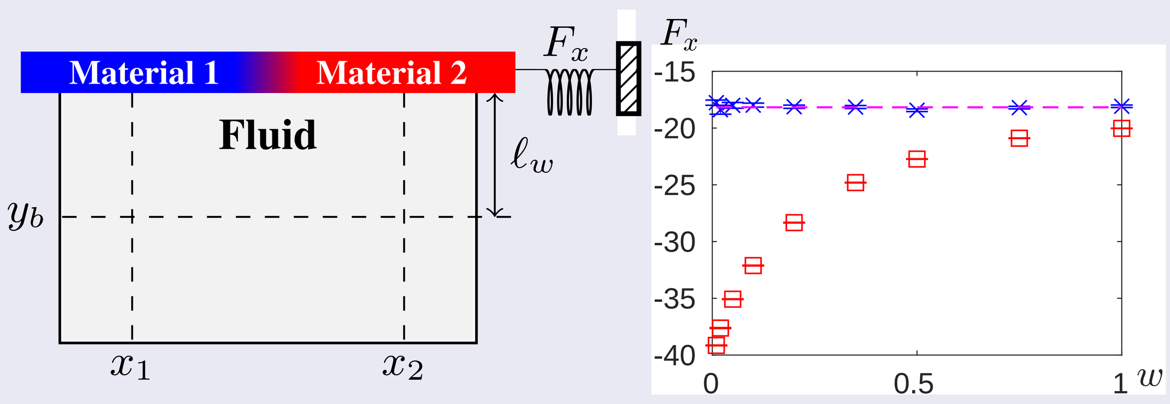

Figure 1: Left: Modified Langmuir setup: a fluid is confined by a mobile upper boundary made of two different materials, modelled by a potential $V(x,y) = \lambda(x)(y-y_w)^4$ for $y > y_w$. The function $\lambda(x)$ interpolates smoothly between the two material stiffnesses λ_1 and λ_2 with a connection of width w. Right: Total tangential force F_x exerted on the upper wall as a function of the width w of the junction.

Langmuir setup of passive particles

$$F_x = \gamma_2 - \gamma_1 \equiv \Delta\gamma$$

with

$$\gamma_k \equiv -\int_{y_w}^{\infty} dy\, kT\rho(x_k, y)$$

calculated in the bulk of the materials, independent of how we glue them.

Langmuir setup of active particles

$$F_x = \gamma_2 - \gamma_1 + F_D = \Delta\gamma + F_D\ ,$$

where we define the bare, equation of state surface tension γ_k as

$$\gamma_k = -\frac{v_0^2}{2\mu D_r}\Big(\int_{y_b}^{\infty} dy\, \rho(x_k,y) - \int_{y_b}^{y_w} dy\, \rho_0\Big)\,.$$

$$F_D = -\frac{1}{\mu}\int_{x_1}^{x_2} dx \int_{y_b}^{\infty} dy\, J_x\ .$$

The drag F_D depends on the details of the junction, and dresses the bare surface tension on the total tangential force on the solid-gas interface.

Flow dresses the bare EOS surface tension

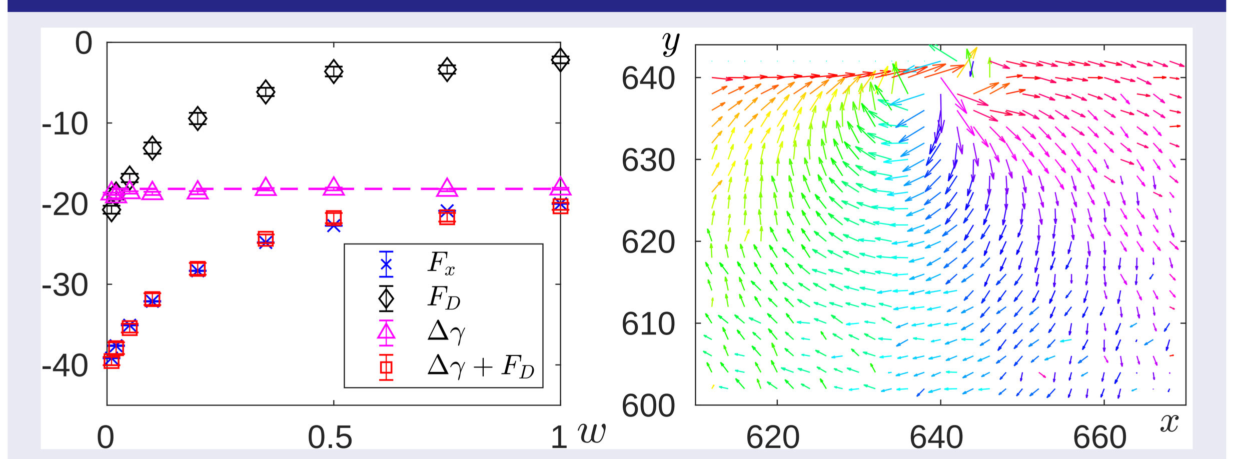

Figure 2: Left: Comparison between F_x and the sum of the bare $\Delta\gamma$ and the junction-dependent drag force F_D. $\Delta\gamma$ measured from the equation above, in the homogeneous bulk region of $V(x,y)$. Right: Map of the current around the junction. The amplitude of the arrows are proportional to $\log|\mathbf{J}/10^{-6}|$; the color encodes its directions.

Unlike pressure, surface tension is made of two contributions: a setup-independent EOS abiding part $\Delta\gamma$ and a current-dependent, setup-specific part F_D.

Bare surface tension and Laplace pressure

Consider gas in a circular container of radius R, the pressure on the wall is modified from the bulk pressure P_b by

$$P \simeq P_b - \frac{1}{R}\Big[\int_0^R \frac{v_0^2\rho_0}{2\mu D_r}dr - \int_0^\infty \frac{v_0^2\rho(r)}{2\mu D_r}dr\Big]\,,$$

$$P_b - P \simeq \frac{\gamma}{R}\,.$$

Thus despite the tangential forces are setup dependent, the bare EOS surface tension still play an important, thermodynamic role: It is related with the Laplace-like pressure in a curved container.

Measurement of the bare surface tension

The junctions on the wall always create currents, but the bare surface tension is still accessible in measurements. We can design a system in which the current doesn't enter the force balance to measure directly the bare surface tension.

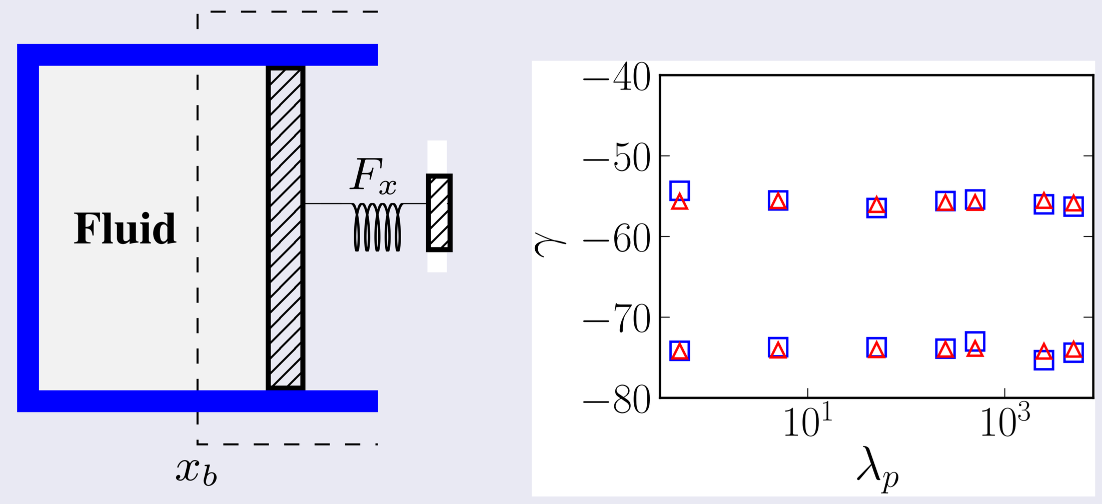

Figure 3: Left: Setup used to directly measure the bare surface tension. Right: The measurement of γ for two different confining materials are shown to be independent of the piston's stiffnesses λ_p. Measuring γ from the force on the piston (blue squares) or from the 'bulk correlator' evaluated at $x = x_b$ (red triangles) lead to consistent results. Upper and lower results correspond to confining materials of stiffness $\lambda = 5$ and $\lambda = 0.05$, respectively. The distance between the two measurements corresponds to $\Delta\gamma$ in Fig. 2.

Conclusion

- Generically, tangential forces at interface (F_x) = bare EOS surface tension $(\Delta\gamma)$ + non-universal contribution due to current (F_D).
- Bare EOS surface tension is related to the Laplace pressure, thus has a thermodynamic role.
- It is possible to measure directly the bare EOS surface tension, if we design a smart setup.

Reference

[1] R. Zakine, Y.F. Zhao, M. Knežević, A. Daerr, Y. Kafri, J. Tailleur, F. van Wijland, Phys. Rev. Lett. (2020).